FIN 300

Time and Money

Lecture 4

TOPICS COVERED

See Textbook Chapter 5

- Present Value, Future Value and Interest
- Compounding
- Compound and Simple Interest
- Discount Rates

PRESENT VALUE, FUTURE VALUE AND INTEREST

- Fundamental trade-off
- The interest rate is the price of borrowing
- For loans, present and future values can be related through the interest rate
- One period loan:

 $ext{Present Value} imes (1 + ext{Interest Rate}) = ext{Future V}$

$100 \times (1 + 10\%) = 110$

• We can rearrange our equation to isolate present value

$$\mathrm{PV} = rac{\mathrm{FV}}{1+\mathrm{r}}$$
 $\$100 = rac{\$110}{(1+10\%)}$

COMPOUNDING

- What happens if the loan continues for multiple periods?
- Imagine the first year has passed and we already have \$110.
- What would the value be after one more year?

$$\mathrm{V}_2 = \mathrm{V}_1 imes (1+r)$$

$$\mathrm{V}_2 = 110 imes (1+0.1)$$

= \$121

We could find the third year by multiplying 121 by (1+r)

- Year by year calculations are not efficient
- We can make this easier by spotting a pattern
- $V_1 = V_0 \times (1+r)$ • $V_2 = V_1 \times (1+r) = V_0 \times (1+r) \times (1+r)$ • $V_2 = V_0 \times (1+r)^2$ $FV_t = PV_0 \times (1+r)^t$

COMPOUND AND SIMPLE INTEREST

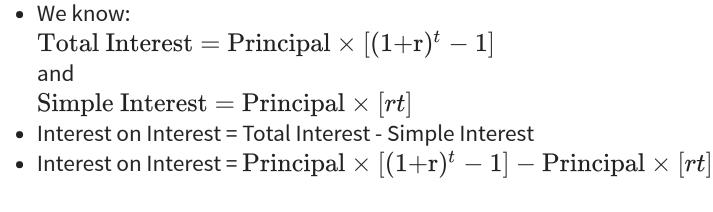
- As the investment grows, the total interest payment increases
- We can dissect the total (compounded) interest:

Compound Interest = Simple Interest + Interest on Interest

- Simple Interest: amount paid on the original principal
- Interest on Interest: amount paid on accrued interest

- Total interest payment: Future Value Principal
- $\mathrm{FV}_t = \mathrm{Principal} \times (1+r)^t$
- FV_t -Principal=Principal× $(1+r)^t$ -Principal
- Total Interest = Principal $imes [(1+r)^t 1]$
- Simple interest is based only on the principal Simple Interest = Principal $\times rt$

How do we calculate interest on interest?



• Interest on Interest = Principal $\times [(1+r)^t - 1 - rt]$

DISCOUNT RATES

- Sometimes we may know the present and future values
- From this, we can figure out an *implied* rate of return
- This is known as the discount rate
- Calculating discount rates requires solving for *r*:

$$rac{\mathrm{FV}}{(1+r)^t} = \mathrm{PV}$$

To get the discount rate, isolate r

$$rac{\mathrm{FV}}{(1+r)^t} = \mathrm{PV}$$
 [multiply by $(1+r)^t$ and divide by PV]

$$rac{\mathrm{FV}}{\mathrm{PV}} = (1+\mathrm{r})^t$$
 [raise to exponent $1/t$]

$$\left(rac{\mathrm{FV}}{\mathrm{PV}}
ight)^{1/t} = (1+\mathrm{r})^{t/t} = 1+r$$
 [subtract 1]

 $\left(rac{\mathrm{FV}}{\mathrm{PV}}
ight)^{1/t} - 1 = r$

Example

Suppose: PV = 10, FV = 45, and t = 10r = ? $\left(\frac{\mathrm{FV}}{\mathrm{PV}}\right)^{1/r} - 1 = r$ $\left(\frac{45}{10}\right)^{1/10} - 1 = r = 0.1623$

SUMMARY

