FIN 300

Time and Money

Lecture 4

TOPICS COVERED

See Textbook Chapter 5

- Present Value, Future Value and Interest
- Compounding
- Compound and Simple Interest
- Discount Rates

PRESENT VALUE, FUTURE VALUE AND INTEREST

- Fundamental trade-off
- The interest rate is the price of borrowing
- For loans, present and future values can be related through the interest rate
- One period loan:

$$\text{Present Value} \times (1 + \text{Interest Rate}) = \text{Future V}$$

$$\$100 \times (1 + 10\%) = \$110$$

- We can rearrange our equation to isolate present value

$$\mathrm{PV} = \frac{\mathrm{FV}}{1 + \mathrm{r}}$$

$$\$100 = \frac{\$110}{(1 + 10\%)}$$

COMPOUNDING

- What happens if the loan continues for multiple periods?
- Imagine the first year has passed and we already have $\$110$.
- What would the value be after one more year?

$$\mathrm{V}_2 = \mathrm{V}_1 \times (1 + r)$$

$$\mathrm{V}_2 = 110 \times (1 + 0.1)$$

$$= \$121$$

We could find the third year by multiplying 121 by (1+r)

- Year by year calculations are not efficient
- We can make this easier by spotting a pattern
- $\mathrm{V}_1 = \mathrm{V}_0 \times (1 + r)$
- $\mathrm{V}_2 = \mathrm{V}_1 \times (1 + r) = \mathrm{V}_0 \times (1 + r) \times (1 + r)$
- $\mathrm{V}_2 = \mathrm{V}_0 \times (1 + r)^2$

$$\mathrm{FV}_t = \mathrm{PV}_0 \times (1 + r)^t$$

COMPOUND AND SIMPLE INTEREST

- As the investment grows, the total interest payment increases
- We can dissect the total (compounded) interest:

Compound Interest = Simple Interest + Interest on Interest

- Simple Interest: amount paid on the **original** principal
- Interest on Interest: amount paid on accrued interest

- Total interest payment: Future Value - Principal
- $\mathrm{FV}_t = \text{Principal} \times (1 + r)^t$
- $\mathrm{FV}_t - \text{Principal} = \text{Principal} \times (1 + r)^t - \text{Principal}$
- $\text{Total Interest} = \text{Principal} \times [(1+\mathrm{r})^t - 1]$

- Simple interest is based only on the principal

  Simple Interest = Principal $\times rt$

How do we calculate interest on interest?

- We know:
  $\text{Total Interest} = \text{Principal} \times [(1+\text{r})^t - 1]$
  and
  $\text{Simple Interest} = \text{Principal} \times [rt]$
- Interest on Interest = Total Interest - Simple Interest
- Interest on Interest = $\text{Principal} \times [(1+\text{r})^t - 1] - \text{Principal} \times [rt]$

- $\text{Interest on Interest} = \text{Principal} \times [(1+\text{r})^t - 1 - rt]$

DISCOUNT RATES

- Sometimes we may know the present and future values
- From this, we can figure out an *implied* rate of return
- This is known as the discount rate
- Calculating discount rates requires solving for r:

$$\frac{\mathrm{FV}}{(1+r)^t} = \mathrm{PV}$$

To get the discount rate, isolate r

$$\frac{\mathrm{FV}}{(1+r)^t} = \mathrm{PV}$$ [multiply by $(1+r)^t$ and divide by PV]

$$\frac{\mathrm{FV}}{\mathrm{PV}} = (1+\mathrm{r})^t$$ [raise to exponent $1/t$]

$$\left(\frac{\mathrm{FV}}{\mathrm{PV}}\right)^{1/t} = (1+\mathrm{r})^{t/t} = 1+r$$ [subtract 1]

$$\left(\frac{\mathrm{FV}}{\mathrm{PV}}\right)^{1/t} - 1 = r$$

Example

Suppose: $PV = 10, FV = 45$, and $t = 10$
$r = ?$

$$\left(\frac{\mathrm{FV}}{\mathrm{PV}}\right)^{1/t} - 1 = r$$

$$\left(\frac{45}{10}\right)^{1/10} - 1 = r = 0.1623$$

SUMMARY

$$\mathrm{PV} = \frac{\mathbf{FV}}{(1+r)^t}$$