FIN 300

Discounted Cash Flows

Lecture 5a

TOPICS COVERED

- Multiple Cash Flows
- Annuities
- Perpetuities
- Growing Annuities and Perpetuities
- Different Types of Loans

MULTIPLE CASH FLOWS

- Investments involve cash flows that are separated in time
- We can add up the value of a series of cash flows over time
- Each cash flow must be correctly discounted

Example

$$\$100 + \$100 = \$200$$

Instead (Assume r = 1%)

$$\$100 + \frac{\$100}{(1 + 0.01)} = \$199.01$$

What if we received $\$100$ for 6 years starting next year?

$$PV = \frac{\$100}{(1+0.01)^1} + \frac{\$100}{(1+0.01)^2} + \frac{\$100}{(1+0.01)^3} + \frac{\$100}{(1+0.01)^4} + \frac{\$100}{(1+0.01)^5} + \frac{\$100}{(1+0.01)^6}$$

$$= \$579.55$$

This is an example of an annuity

ANNUITIES

- A stream of cash flows over a finite period
- Fixed annuities have a level stream

  $CF_1 = CF_2 = \cdots = CF_t$

$$PV_{Annuity} = CF \times \left\{ \frac{1 - [1/(1+r)^t]}{r} \right\}$$

Example

What if we received $\$100$ for 6 years starting next year?

$$PV_{Annuity} = \$100 \times \left\{ \frac{1 - [1/(1+0.01)^6]}{0.01} \right\}$$

$$= \$579.55$$

Example

What if we received $\$100$ for 600 years starting next year?

$$PV_{Annuity} = \$100 \times \left\{ \frac{1 - [1/(1+0.01)^{600}]}{0.01} \right\}$$

$$= \$9,974.46$$

What would the value be if this continued forever?

$$\lim_{t \to \infty} CF \times \left\{ \frac{1 - [1/(1+r)^t]}{r} \right\}$$

$$= \frac{CF}{r}$$

PERPETUITIES

- Eternal annuities
- Not typically observed in practice
- An approximation for a long term annuity
- The math will be reused for stock valuation

$$PV_{Perpetuity} = \frac{CF}{r}$$

Example

What is the present value of receiving $\$100$ every year forever, starting one year from today? Assume $r = 1\%$.

$$PV_{Perpetuity} = \frac{\$100}{0.01} = \$10,000$$

GROWING ANNUITIES AND PERPETUITIES

- What happens if the payments increase each year?
- We can adapt the formula to growth:

$$PV_{\text{Growing Annuity}} = CF \times \left\{ \frac{1 - \left(\frac{1+g}{1+r}\right)^t}{r-g} \right\}$$

- This work for perpetuities as well:

$$PV_{\text{Growing Perpetuity}} = \frac{CF}{r-g}$$

- For this to work: $r > g$

DIFFERENT TYPES OF LOANS

- Pure Discount
- Interest Only
- Amortized Loan

SUMMARY

- PV of a cash flow series
- Annuities and Perpetuities
- Growing Annuities and Perpetuities
- Different Types of Loans

FIN 300

Interest Rates and Inflation

Lecture 5b

TOPICS COVERED

- Annualized Interest Rates
- Quoted vs Effective Interest Rates
- Inflation

ANNUALIZED INTEREST RATE

- Measuring interest rates is dependent on the time frame
- To get annual rates from shorter periods, interest must be compounded
- Consider a quarterly interest rate: $r_q = 2\%; \; r_a = ?$

$\$1$ today $\rightarrow$? in one year

$$(1+0.02) \times (1+0.02) \times (1+0.02) \times (1+0.02) = 1.0824$$

$$(1+r_q) \times (1+r_q) \times (1+r_q) \times (1+r_q) = 1+r_a$$

$$(1+r_q) \times (1+r_q) \times (1+r_q) \times (1+r_q) = 1+r_a$$

$$(1+r_q)^4 = 1+r_a$$

$$r_a = (1+r_q)^4 - 1$$

The General Case:

$$r_a = \left(1 + r_{\frac{1}{m}}\right)^m - 1$$

m is the number of compounds per year

Example:

In annual terms, how much is a daily rate of 0.01%?

$$r_a = (1 + r_{\frac{1}{m}})^m - 1 = (1 + r_{\frac{1}{365}})^{365} - 1$$

$$r_a = (1 + 0.0001)^{365} - 1 = \mathbf{0.037} = \mathbf{3.7\%}$$

QUOTED VS EFFECTIVE RATES

- Quoted rates understate the true interest rate
- True rate implied by number of periods compounded
- Interest on interest makes a difference

A more accurate rate of interest:

Effective Annual Rate (EAR)

QUOTED RATE TO EAR

- Quoted Rate: 12%, compounded semiannually
- Every 6 months: interest rate is 6%
- EAR is the annualized return

$$EAR = (1 + 0.06)^2 - 1 = 0.124 = 12.4\%$$

$$EAR = 12.4\% > 12\% = Quoted\ Rate$$

$$EAR = \left(1 + \frac{\text{Quoted Rate}}{m}\right)^m - 1$$

m is the number of compounding periods

$$EAR = \left(1 + \frac{0.12}{2}\right)^2 - 1$$

$$= 12.4\%$$

INFLATION

- Investments usually return currency
- The value of currency usually decreases
- Nominal returns are the observed returns in currency
- Real returns remove the effects of inflation

Fisher Effect:

$$1 + R = (1 + r) \times (1 + h)$$

Nominal return: R Real return: r Inflation: h

$$1 + R = (1 + r) \times (1 + h)$$

$$1 + R = 1 + r + h + r \times h$$

$$R = r + h + r \times h$$

$$R \approx r + h$$

Nominal returns are approximately equal to real returns plus inflation.

SUMMARY

- Annualized Interest Rates
- Quoted Rate to EAR
- Inflation