FIN 300

Risk and Return

Lecture 10

TOPICS COVERED

- Expected Return
- Variance and Standard Deviation
- Portfolios
- Portfolio Variance
- Diversification and Risk

EXPECTED RETURN

- Point-estimate of average belief
- Many possible versions of the future

$$E(R) = \sum_{s=1}^{S} Pr(s) \times R_s$$

  - S possible states of the world (s)
  - R_s is the return to the stock in state s
  - $Pr(R_s)$ probability of R_s

EXAMPLE

Suppose there was a 50% probability that a stock has a return of 5% and a 50% probability the stock had a return of 15%. The expected return $E(R)$ would be:

$$E(R) = 50\% \times 5\% + 50\% \times 15\% = 10\%$$

$$E(R) = 0.5 \times 0.05 + 0.5 \times 0.15 = 0.1$$

We could also have calculated the expected return in a similar way if there was a 99% probability of a 5% return and a 1% probability of a 15% return:

$$E(R) = 0.99 \times 0.05 + 0.01 \times 0.15 = 0.051$$

Consider three states of the world (for next year): good, bad, and ugly. The probability that we have a good state is 50%. There is a 25% probability for bad and 25% for ugly. Suppose our stock earns 30% in the good state, 0% in the bad state, and -30% in the ugly state. What is the expected return on our stock?

$$E(R) = 0.5 \times 0.30 + 0.25 \times 0 + 0.25 \times -0.30 = 0.075$$

VARIANCE

- Variance is a measure of dispersion for the possible future returns
- This tells us, overall, how far away the various possible outcomes are from the expected value

$$Var(R) = \sigma_R^2 = \sum_{s=1}^{S} Pr(s) \times (R_s - E(R))^2$$

EXAMPLE

From our previous example, we can calculate the value of the return's variance:

$$\begin{aligned}\sigma_R^2 = \ & 0.5 \times (0.30 - 0.075)^2 \\ & + 0.25 \times (0 - 0.075)^2 \\ & + 0.25 \times (-0.30 - 0.075)^2 \\ = \ & 0.0619\end{aligned}$$

STANDARD DEVIATION

To calculate standard deviation, we simply take the square root of the variace:

$$\sigma_R = \sqrt{\sigma_R^2} = \left[\sum_{s=1}^{S} Pr(R_s) \times (R_s - E(R))^2\right]^{\frac{1}{2}}$$

From our example:

$$\sqrt{0.0619} = 0.249$$

PORTFOLIOS

- Collection of investments
- Multiple assets
- Portfolio weights are asset value divided by total value

$$w_i = \frac{Value_i}{\sum_{i=1}^{N} Value_i}$$

EXAMPLE

Suppose I had money invested in two assets: $\$300$ in company X and $\$100$ in company Y.

The total value of my portfolio would be $\$400$.

My weight in company X would be $\frac{\$300}{\$400}$, or 0.75.

$$w_X = 0.75\,;\, w_Y = 0.25$$

EXPECTED RETURN

- Relatively straightforward
- Sum of each stock's weight multiplied by expected return:

$$E(R_p) = \sum_i^N w_i \times E(R_i)$$

EXAMPLE

From our example, suppose companies X and Y had expected returns of 0.125 and 0.06 respectively. Our portfolio's expected return would be:

$$E(R_p) = 0.75 \times 0.125 + 0.25 \times 0.06 = 0.1088$$

PORTFOLIO VARIANCE

- Slightly more complicated
- Volatility can be partially cancelled out by diversification
- Intuition:
  - 5 friends play poker
  - Each person's balance fluctuates
  - Total cash at table is constant
  - Very negative correlation in returns here

- Calculating portfolio variance requires that we know an additional piece of information
- Covariance
- Calculation of covariance:

$$\sigma_{x,y} = \sum_s^S Pr(s) \left[(R_{s,x} - E(R_x)) \times (R_{s,y} - E(R_y)) \right]$$

EXAMPLE (COVARIANCE)

Let's say that there are two states of the world "Bull Market" and "Bear Market" with equal probability (0.5). Let's say we also know the returns to stocks X and Y in these two states:

	R_X	R_Y
Bull Market	.35	.08
Bear Market	-.10	.04

First, we should calculate $E(R_X)$ and $E(R_Y)$:

$$E(R_X) = 0.5 \times 0.35 + 0.5 \times (-0.10) = 0.125$$

$$E(R_Y) = 0.5 \times 0.08 + 0.5 \times 0.04 = 0.06$$

At this point, we can apply the covariance formula directly:

$$\begin{aligned}\sigma_{X,Y} = {} & 0.5[(0.35 - 0.125) \times (0.08 - 0.06)] \\ & + 0.5[(-0.10 - 0.125) \times (0.04 - 0.06)]\end{aligned}$$

$$= 0.0045$$

PORTFOLIO VARIANCE FORMULA

The variance portfolio of a portfolio with two stocks (X and Y) can be calculated in the following way:

$$\sigma_P^2 = w_X^2 \sigma_X^2 + w_Y^2 \sigma_Y^2 + 2 w_X w_Y \sigma_{x,y}$$

Returning to our example with stocks X and Y, we'll also need to calculate their respective return variances:

$$\sigma_X^2 = 0.5 \times (0.35 - 0.125)^2 + 0.5 \times (-0.10 - 0.125)^2 = 0.050$$

$$\sigma_Y^2 = 0.5 \times (0.08 - 0.06)^2 + 0.5 \times (0.04 - 0.06)^2 = 0.0004$$

Once we know, $\sigma_X^2, \sigma_Y^2, \sigma_{x,y}$ and the portfolio weights (0.75, and 0.25 from our example), we can finally calculate the portfolio's return variance:

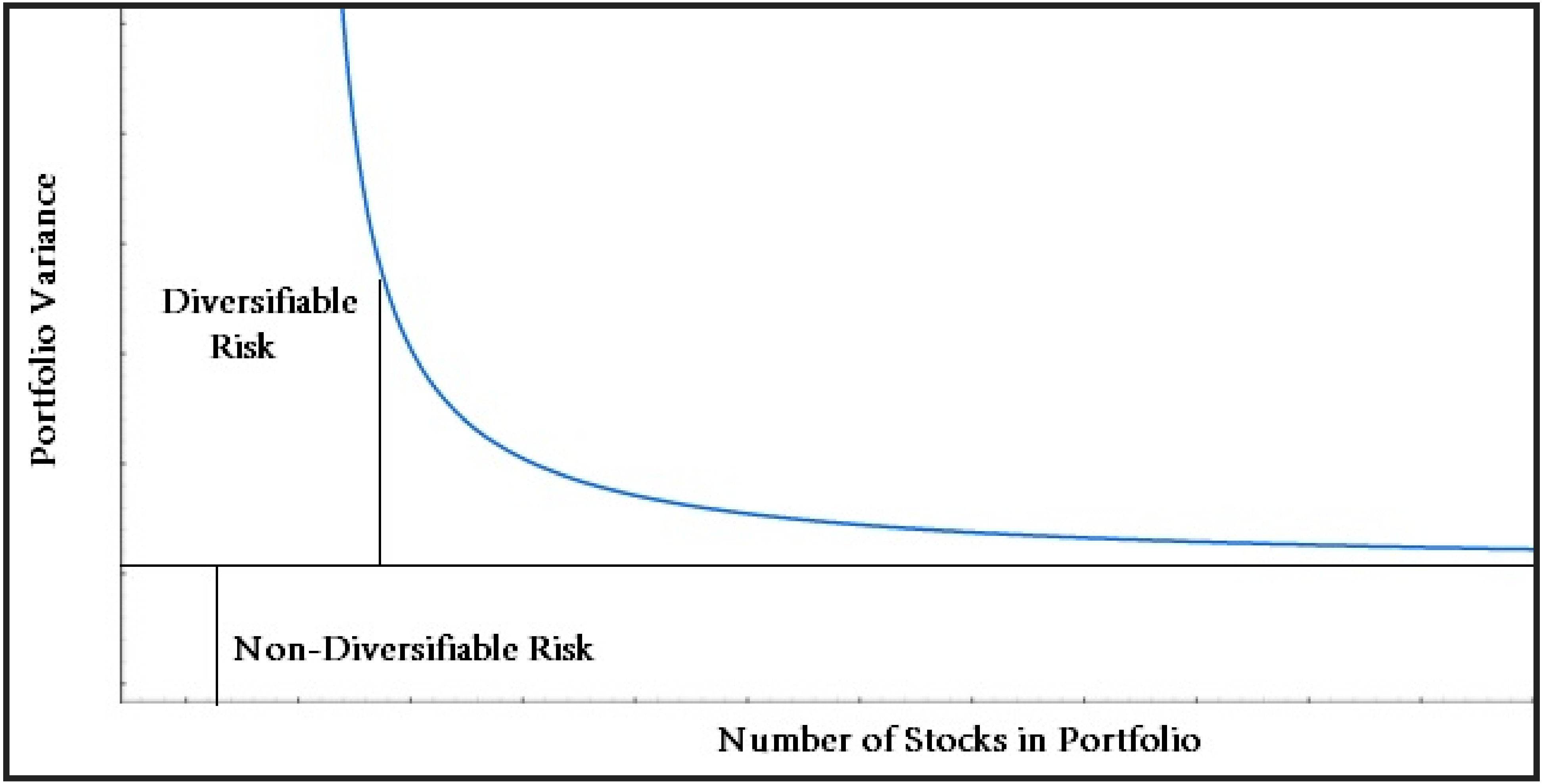
$$\sigma_P^2 = (0.75^2) \times (0.0506) + (0.25^2) \times (0.0004) \\ + 2 \times (0.75) \times (0.25) \times (0.0045)$$

$$\sigma_P^2 = 0.030189; \sigma_P = \sqrt{0.030189} = 0.17375$$

DIVERSIFICATION AND RISK

- Don't put all your eggs in one basket
- Not perfectly correlated
- Generalize the portfolio variance formula:

$$\sigma_P^2 = \sum_{i=1} w_i^2 \sigma_X^2 + \sum_{i=1} \sum_{i \neq j} w_i w_j \sigma_{i,j}$$


Portfolio Variance
Diversifiable
Risk
Non-Diversifiable Risk
Number of Stocks in Portfolio

- Diversifiable Risk
  - Idiosyncratic Risk
  - Non-Systematic
- Non Diversifiable Risk
  - Market Risk
  - Systematic Risk